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1. (a) (i) Use Taylor's theorem to expand $\sin\left(\frac{\pi}{4} + h\right)$ in ascending powers of h as far as the term in h^3.

(ii) Hence determine the value of Sin 47°, giving the answer correct to five decimal places.

(9 marks)

(b) (i) Determine the Maclaurin's series expansion for $f(x) = \ln(1 + 3x)$, in ascending powers of x up to and including x^4.

(ii) Hence evaluate $\int_0^1 \frac{1}{x}\ln(1+3x)dx$. (11 marks)

2. (a) Given the matrices

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Find $A^2 - 4B$ (4 marks)

(b) Solve the equation:

$$\begin{vmatrix} x & 3 & 2 \\ 1 & 1 & x \\ x & 1 & 2 \end{vmatrix} = 4$$

(4 marks)

(c) Use inverse matrix method to solve the following simultaneous equations:

$$x + 2y + 3z = 6$$
$$2x + y + z = 5$$
$$3x + y - 2z = 1$$

(12 marks)

3. (a) Determine the inverse Laplace transform of $F(s) = \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$.

(8 marks)

(b) Use Laplace transforms to solve the differential equation:

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 5e^t$, given that when $t = o$, $x = 2$ and $\frac{dx}{dt} = 1$. (12 marks)

4. (a) (i) A series circuit contains an inductor and resistor. The circuit is connected to a constant voltage source E at time $t = 0$. If initially the circuit was inactive, determine the expression for current for $t > 0$.

(ii) Hence obtain the expression for steady state current.

(8 marks)

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(b) Use D - operator method to solve the following differential equation $(D^2 + 4D + 3)y = x^3 + x^2 + 2$. (12 marks)

5. (a) Given that $u = \frac{x - y}{x + y}$

Show that $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} - \frac{2d^2u}{dxdy} = 0$ (7 marks)

(b) The power consumed in an electrical resistor is given by $P = \frac{E^2}{R}$ Watts, where E is the voltage drop across the resistor in Volts and R is the resistance of the resistor in ohms. Use partial derivatives to approximate change in power when E increases by 4 percent and R decreases by 0.05 percent, if the original values of E and R are 100 Volts and 10 ohms respectively. (7 marks)

(c) Determine the stationary values of $z = x^3 - 6xy + y^3$. (6 marks)

6. (a) (i) Given the vectors

$\underset{\sim}{A} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

$\underset{\sim}{B} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{C} = 4\underset{\sim}{i} + p\underset{\sim}{j} + 2\underset{\sim}{k}$

where p is a constant.

Find the value of p so that vectors $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{C}$ are co-planar.

(ii) Find a unit vector perpendicular to vectors $\underset{\sim}{M} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{N} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$. (7 marks)

(b) Given that $\phi(x,y,z) = 3x^2y + 4y^2z^3$. Determine the:

(i) directional derivative of ϕ in the direction of vector $\underset{\sim}{B} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ at the point (2,1,1).

(ii) Div. Grad ϕ at the point (3, -1, 2).

(8 marks)

(c) Vector $\underset{\sim}{A} = x^2y\underset{\sim}{i} + xyz^2\underset{\sim}{j} + x^3y\underset{\sim}{k}$. Determine curl $\underset{\sim}{A}$ at the point (2, 1, 2). (5 marks)

7. (a) Solve the differential equation $\frac{dy}{dx} + y \cot x = \sin x$. (5 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 6x + 4\cos x$, given that $y = 2$, $\frac{dy}{dx} = 4$ when $x = 0$. (15 marks)

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8. (a) A radar unit is used to measure the speed of automobiles on an express way during rush hour traffic. The speeds of individual automobiles are normally distributed with a mean of 62 kph.
Determine the:

(i) standard deviation of all speeds if 3% of the automobiles travel faster than 72 kph.

(ii) Percentage of cars that travels with speeds less than 58 kph.

(iii) 95th percentile for the variable "speed".

(8 marks)

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(b) The lifetime of an electrical bulb in years is represented by a continuous random variable T, defined by the probability density function,

$$f(t) = \begin{cases} \frac{C^2}{2} e^{-ct} & ; \ t \geq 0 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

where c is a constant. Determine the:

(i) value of constant c;
(ii) expected lifetime;
(iii) median;
(iv) cumulative distribution function (c.d.f) F(t).

(12 marks)