Name:______________________________ Index No.: ______________________

2521/102
2601/103
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ENGINEERING MATHEMATICS I
June/July 2015
Time: 3 hours

Candidate's Signature:________________

Date:______________________

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)
MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

Write your name and index number in the spaces provided above.
Sign and write the date of the examination in the spaces provided above.
You should have Mathematical tables / Scientific calculator for this examination.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the spaces provided in this question paper.*
All questions carry equal marks.
Maximum marks to each part of a question are as shown.
Do ***NOT*** *remove any pages from this booklet.*
Candidates should answer the questions in English.

For Examiner's Use Only

Question	1	2	3	4	5	6	7	8	TOTAL SCORE
Candidate's Score									

This paper consists of 20 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

2015 7/2015

1. (a) Prove the identities:

 (i) $\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$;

 (ii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$. (7 marks)

 (b) (i) Express $\operatorname{sech}^{-1} x$ in logarithmic form;

 (ii) Given that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, find the real root of the equation $\operatorname{sech}^{-1} x = \sinh^{-1} x$. (13 marks)

2. (a) Prove the identity:

2 sin θ cos θ

$$\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$ (4 marks)

 (b) Given that A, B and C are angles of a triangle, prove that

 $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$ (7 marks)

 (c) (i) Express $5\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 \le \alpha \le 90°$;

 (ii) Hence, solve the equation $5\sin\theta - 12\cos\theta = 6$ for $0 \le \theta \le 360°$. (9 marks)

3. (a) Solve the equation $2x^2 - 9x + 9 = 0$, by factorization. (5 marks)

 (b) The roots of the equation $x^2 + 6x + q = 0$ are α and $\alpha - 1$. Determine the value of q. (5 marks)

 (c) The roots of the equation $x^2 + 7x + 3 = 0$ are α and β. Without solving the equation, form an equation whose roots are $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$. (10 marks)

4. (a) Find the middle term in the binomial expansion of $(2x+3)^8$, and determine its value when $x = \dfrac{1}{12}$. (6 marks)

 (b) Expand $(1-3x)^{-\frac{1}{2}}$ as far as the term in x^3 and determine the range of values of x for which the expansion is valid. (4 marks)

 (c) (i) If x is so small that its fourth and higher powers may be neglected, show that $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)} = a - bx^2$, and determine the values of a and b;

 (ii) Hence, by putting $x = \dfrac{1}{16}$ in the result in (c) (i) above, prove that $17^{\frac{1}{4}} + 15^{\frac{1}{4}} = 3.9985$ approximately. (10 marks)

j = sin

1. (a) Simplify the expressions:

(i) $$\frac{(1-x)^{\frac{1}{2}}-(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}$$

(ii) $$\frac{\log 125-\log 25+\log 5}{\log 625+\frac{1}{2}\log 25}$$

$$\frac{3\log 5 - 2\log 5 + \log 5}{4\log 5 + \log 5}$$

$$\frac{\log 5^3 - \log 5^2 + \log 5}{\log 5^4 + \log 5}$$

$$\frac{\log 5(3-2+1)}{\log 5(4+1)} = \frac{2}{5}$$

(7 marks)

(b) Solve the equation:

$$4^x + 1 = 3 + 2^x$$

(5 marks)

(c) The application of Kirchoff's laws to a d.c. circuit yielded the simultaneous equations:

$$I_1 - 2I_2 + I_3 = 0$$

$$-2I_1 + 3I_2 + 2I_3 = 2$$

$$3I_1 + 4I_2 - 3I_3 = 14$$

Where I_1, I_2 and I_3 are currents in amperes. Use elimination method to solve the equations. (8 marks)

2. (a) Find the coefficient of x^6 in the binomial expansion of $(3x+2y)^{10}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{1}{3}$. (5 marks)

(b) (i) Determine the first four terms in the binomial expansion of $(3+4x)^{-\frac{1}{2}}$, and state the values of x for which the expansion is valid.

(ii) Use the binomial theorem to expand $\left(1+\frac{1}{4}x\right)^{\frac{1}{3}}$ as far as the term in x^3. Hence determine the value of $\sqrt[3]{65}$, correct to four decimal places. (9 marks)

(c) Solve the equation:

$$3^{2x+1} - 7(3^x) + 2 = 0$$

(6 marks)

3. (a) Given the complex numbers $z_1 = 2+3j$, $z_2 = 1+2j$ and $z_3 = 3-4j$, express

$z_1 + \dfrac{z_2 z_3}{z_2 + z_3}$ in polar form.

(8 marks)

(b) One root of the equation $z^3 + 4z^2 + kz + 8 = 0$ is $-1 + j\sqrt{3}$. Determine the:

(i) value of k;
(ii) other roots.

(6 marks)

(c) Solve the equation:

$$z^3 - 1 + j\sqrt{3} = 0$$

(6 marks)

4. (a) Given $y = \frac{x}{1-x}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Use implicit differentiation to determine the equations of the:

(i) tangent;
(ii) normal

to the curve $x^3 + y^2 + 3xy - 2x + 6y + 9 = 0$ at the point $(1, -1)$. (10 marks)

(c) Determine the stationary points of the curve $f(x) = x^3 + 15x^2 + 27x + 2$, and state their nature. (5 marks)

5. (a) (i) Evaluate the indefinite integral

$$\int \frac{\sin 2x}{1+\sin^2 x}\,dx$$

(ii) Show that $\int_0^1 \frac{x}{(x+1)(x^2+x+1)}\,dx = \ln\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6\sqrt{3}}$

(12 marks)

(b) Use the integration to determine the length of the curve $y = \frac{1}{3}x^{\frac{3}{2}}$ between the points $x = 0$ and $x = 4$. (8 marks)

6. (a) Prove the trigonometric identities:

(i) $$\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$$

(ii) $$\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$$

(9 marks)

(b) Solve the equation:

$3\sin^2\theta + 5\cos\theta = 5$, for values of θ between $0°$ and $360°$ inclusive.

(5 marks)

(c) (i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.

(ii) Hence solve the equation:

$4\cos\theta + 3\sin\theta = 5$ for values of θ between 0° and 180° inclusive. (6 marks)

7. (a) Given $u = \dfrac{x - 3y}{x + 3y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$ (5 marks)

(b) The radius of a right circular cone is increasing at a rate of 18 cm/s while its height is decreasing at a rate of 25 cm/s. Determine the rate of change of the volume of the cone when the radius is 120 cm and the height is 140 cm. (4 marks)

(c) Locate the stationary points of the function $z = x^3y + 12x^2 - 8y + 2$, and determine their nature. (11 marks)

8. (a) Prove the identities:

(i) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$;

(ii) $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

(7 marks)

(b) (i) Express $\tanh^{-1} x$ in logarithmic form.

(ii) Hence determine the value of $\tanh^{-1}\left(\frac{1}{2}\right)$, correct to four decimal places. (8 marks)

(c) Solve the equation $3\cosh^2 x - 7\sinh x - 1 = 0$. (5 marks)

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