

2601/103

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ENGINEERING MATHEMATICS I

June/ July 2016

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)
MODULE I**

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Drawing instruments;

Mathematical tables/ non-programmable scientific calculator.

*This paper consists of **EIGHT** questions.*

*Answer any **FIVE** questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Solve the equations:

(i) $5^{6x+1} \times 25^{x-7} = 125;$ (4 marks)

(ii) $\log_3 (x+2)^2 = 2$ (4 marks)

(b) Convert:

(i) $r = 4(1 + 2\sin 2\theta)$ to cartesian form.

(ii) $xy = 3$ to polar form. (6 marks)

(c) Three currents I_1, I_2 and I_3 in amperes flowing in an electric circuit satisfy the following simultaneous equations:

$$3I_1 + 2I_2 + 5I_3 = 2$$

$$3I_1 + 3I_2 - 2I_3 = 4$$

$$2I_1 - 5I_2 - 3I_3 = 14$$

Use elimination method to determine the values of the three currents. (6 marks)

2. (a) Prove the identity $\frac{\tan\theta + \sec\theta}{\sec\theta(1 + \frac{\tan\theta}{\sec\theta})} = 1$. (4 marks)

(b) If $\sin A = \frac{3}{5}$ and $\cos B = \frac{6}{10}$, where A and B are acute angles, determine:

(i) $\sin (A - B);$

(ii) $\cot 2A.$

(6 marks)

(c) Given that $8 \cos \theta + 36 \sin \theta = R \sin (\theta + \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$:

(i) find the values of R and α ;

(ii) hence, solve the equation $8 \cos \theta + 6 \sin \theta = 6$ for $0^\circ \leq \theta \leq 360^\circ$.

(10 marks)

3. (a) A committee of 5 is to be chosen from 7 men and 6 women. Find the number of ways in which the committee can be formed so that it contains at least 3 men. (5 marks)
- (b) (i) Prove that if x^3 and higher power can be neglected, $\sqrt{\frac{1+3x}{1-3x}} = 1 + 3x + \frac{9}{2}x^2$.
- (ii) Hence, by letting $x = \frac{1}{9}$ in (i) above, show that $\sqrt{2} = 1\frac{11}{25}$. (9 marks)
- (c) The resonant frequency of a series electric circuit is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$, where L is the inductance and C is the capacitance. If L increases by 2.4% and C decreases by 0.7%, determine using the binomial theorem the percentage change in resonant frequency f_r , correct to one decimal place. (6 marks)
4. (a) Find the inverse function of $f(x) = \frac{-2}{x-5}$. (4 marks)
- (b) (i) Show that $\tanh^{-1}x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$;
- (ii) Hence, determine $\tanh^{-1}0.71$, correct to four decimal places. (8 marks)
- (c) Solve the equation $2 \sinh x + 3 \cosh x = 5$, correct to four decimal places. (8 marks)
5. (a) Given that $Z_1 = 1 + j2$, $Z_2 = 2 - j3$ and $Z_3 = -4 + j12$, determine:
- (i) $3Z_1 + Z_2 - Z_3$;
- (ii) $Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$. (7 marks)
- (b) Use Demoivre's theorem to express $\cos^6 \theta$ in terms of the cosines of multiples of θ . (6 marks)
- (c) (i) If $Z = x + jy$, show that the locus defined by $\arg\left\{\frac{Z+2}{Z}\right\} = \frac{\pi}{4}$ is a circle.
- (ii) Hence, determine its centre and radius. (7 marks)

6. (a) Find $\frac{dy}{dx}$ given that:
- (i) $y = x^3 \cos^3 2x$
 - (ii) $xy^3 + y^3x^3 + 4 = 0$
 - (iii) $x = 4 \sec \theta, y = 3 \tan \theta$
- (8 marks)
- (b) If $y = 8x^2e^{-x}$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 16e^{-x}$. (4 marks)
- (c) Given that $f(x) = 2x^3 - \frac{15}{2}x^2 + 9x + 3$, find the:
- (i) coordinates of the turning point;
 - (ii) hence, determine their nature.
- (8 marks)
7. (a) Given that $Z = 2 \cos(4x + 5y)$, show that $4\frac{d^2Z}{dy^2} - 5\frac{d^2Z}{dx^2} = -10Z$. (5 marks)
- (b) The time of oscillation t of a pendulum is given by $t = 2\pi\sqrt{\frac{L}{g}}$. Use partial differentiation to determine the percentage change in t , if L is increasing at 0.3% and g is decreasing at 0.2%. (6 marks)
- (c) Locate the stationary point of the function $f(x,y) = 2x + 2y - 2xy - 2x^2 - y^2 + 4$ and determine their nature. (9 marks)
8. (a) Evaluate the integrals:
- (i) $\int \frac{4x^2 - 7x + 13}{(x-2)(x^2+1)} dx;$
 - (ii) $\int x^4 \ln 2x dx;$
 - (iii) $\int_0^1 \frac{1}{\sqrt{3-2x-x^2}} dx.$
- (12 marks)
- (b) Find the area bounded by the curve $y = 2x^2 + 3x - 4$, the x -axis and the ordinates at $x = 2$ and $x = 4$. (3 marks)
- (c) Determine the root mean square value of the function $y = 200 \sin 250\pi t$, between the ordinates $t = 0$ and $t = \frac{1}{100}$, correct to two decimal places. (5 marks)

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