Name: Segn. Index No.: ______

2521/102 2601/103 2603/103
2522/102 2602/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2013
Time: 3 hours

Candidate's Signature: ______

Date: ______



THE KENYA NATIONAL EXAMINATIONS COUNCIL

# DIPLOMA IN ELECTRICAL AND ELECTRONICS ENGINEERING
# (POWER OPTION)
# (TELECOMMUNICATION OPTION)
# (INSTRUMENTATION OPTION)
# MODULE I

ENGINEERING MATHEMATICS I

**3 hours**

## INSTRUCTIONS TO CANDIDATES

*Write your name and index number in the spaces provided above.*
*Sign and write the date of the examination in the spaces provided above.*
*You should have a Scientific non-programmable calculator for this examination.*
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the spaces provided in this question paper.*
*All questions carry equal marks.*
*Maximum marks to each part of a question are as shown.*
*Do **NOT** remove any pages from this booklet.*
***Candidates should answer the questions in English.***



**For Examiner's Use Only**

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | TOTAL SCORE |
|---|---|---|---|---|---|---|---|---|---|
| Candidate's Score | | | | | | | | | |

**This paper consists of 20 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**



**Turn over**

1. (a) Prove the identity:

$$\frac{\tan\theta + \cot\theta}{\sec\theta \operatorname{cosec}\theta} = 1.$$ (3 marks)

(b) Solve the equation:

$\cos 2\theta + 3 = 5\cos\theta$ for $0° \leq \theta \leq 360°$. (7 marks)

(c) Given that angles A, B and C are included angles of a triangle, prove that:

$$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}.$$ (10 marks)

2. (a) Solve the following equations:

(i) $\log_4(x+3) + \log_4(2-x) = 1$;

(ii) $5^{2x+2} = 3^{5x-1}$. (8 marks)

(b) Solve for $x$ in the equation:

$3^{2x} + 3^{x+1} - 4 = 0$. (4 marks)

(c) Use the elimination and/or substitution method to solve the following simultaneous equation:

$$\frac{1}{x} - \frac{2}{y} - \frac{2}{z} = 0$$
$$\frac{2}{x} + \frac{3}{y} + \frac{1}{z} = 1.$$
$$\frac{3}{x} - \frac{1}{y} - \frac{3}{z} = 3$$ (8 marks)

3. (a) Given the complex numbers

$z_1 = 3 + 5j$
$z_2 = 4 - 6j$
$z_3 = 5 + 2j$

Determine in the form $a + bj$:

(i) $z_1 z_2$;

(ii) $z$ if $\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$. (7 marks)

(b) (i) Express $\sin^4\theta$ in terms of cosines of multiples of $\theta$;

(ii) Find all the solutions to the equation:

$z^4 = 2 + 2\sqrt{3}j$, giving the answers in the form $a + bj$. (13 marks)

4. (a) Convert the polar equation $r = 4a\cot\theta\operatorname{cosec}\theta$ into a Cartesian equation. (4 marks)

(b) (i) Given that $A\cosh x + B\sinh x = 3e^x - 4e^{-x}$ where A and B are constants, determine the values of A and B;

(ii) Find the logarithmic form of $\cosh^{-1}x$. (8 marks)

(c) Determine the equations of the tangents to the circle $x^2 + y^2 - 4x - 2y - 8 = 0$ which are parallel to the line $3x + 2y = 0$. (8 marks)

5. (a) (i) Prove that, if $x$ is so small that its cube and higher powers can be neglected

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{1}{2}x^2.$$

(ii) By taking $x = \frac{1}{9}$, prove that $\sqrt{5}$ is approximately equal to $\frac{181}{81}$. (10 marks)

(b) Use binomial theorem to calculate $\sqrt{0.998}$ correct to four significant figures. (3 marks)



(c) The load that can be supported by a beam is given by the formula $F = \frac{kbd^3}{L}$, where b = breadth, d = depth, L = length of the beam and k is a constant. Use binomial theorem to approximate the percentage increase in load that the beam can support when b is increased by 2%, d is increased by 4% and the length is reduced by 5%. (7 marks)

6. (a) Differentiate the following functions:

(i) $\log_e\left(\frac{1+x}{1-x}\right)$;

(ii) $\sin^{-1}\left(\frac{5}{3}x\right)$. (8 marks)

(b) A curve is defined by the parametric equations $x = \sin^2 t$ and $y = \cos t$. Determine the value of $\frac{dy}{dx}$ at the point of $t = \frac{\pi}{3}$. (5 marks)

(c) A particle P moves in a straight line. After t seconds, the displacement in metres of P from a fixed point O on the line is given by $s = t^3 - 2t^2 + 4t$. Calculate the:

(i) distance between P and O given that the time $t = 2$;

(ii) times at which the velocity of P equals 4 metres per second. (4 marks)

(d) Determine the point of inflexion on the curve $y = x^3 - 3x^2 - 2$. (3 marks)

7. (a) Evaluate the following integrals:

(i) $\int_1^2 \frac{x+1}{x(x^2+1)}dx$;

(ii) $\int_0^{\frac{\pi}{4}} \sin 5x \sin 3x \, dx$. (12 marks)

(b) (i) Sketch the region bounded by the graphs $y = (x+1)^2$ and $y = 3x + 3$.

(ii) Use integration to determine the area of the region in (a) (i). (8 marks)



8. (a) Find the slope of the tangent to the curve $(x+3)^2 - 4(y-2)^2 = 9$ at the point (2, 4). (4 marks)

(b) Given that $z = \frac{1}{\sqrt{x^2 + y^2}}$, determine:

(i) $\frac{\delta^2 z}{\delta x^2}$;

(ii) $\frac{\delta^2 z}{dy^2}$;

(iii) $\frac{\delta^2 z}{dxdy}$. (10 marks)

(c) Determine the stationary values of the function $f(x, y) = x^3 - 3x^2 - 4y^2 + 2$. (6 marks)