1. (a) Given the matrices $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ -6 & 1 & -2 \end{bmatrix}$, determine $(BA)^T$. (4 marks)

(b) Find the inverse of the matrix

$$= \begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}.$$

(7 marks)

(c) Current I_1, I_2 and I_3 in an electrical circuit satisfy the simultaneous equations:

$$I_1 + I_2 + I_3 = 6$$
$$I_1 - 2I_2 + 2I_3 = 5$$
$$3I_1 + I_2 + I_3 = 8$$

Use Cramer's rule to determine the values of the currents. (9 marks)

2. (a) Write down the middle term in the binomial expansion of $(3x + 2y)^{10}$, and determine its value when $x = \frac{1}{4}$ and $= \frac{1}{9}$. (7 marks)

(b) Use the binomial theorem to determine the value of $\frac{1}{(1.02)^7}$ correct to four decimals. (5 marks)

(c) (i) Find the first **four** terms in the binomial expansion of $(1 - 8x)^{\frac{1}{2}}$ in ascending powers of x.

(ii) By substituting $x = \frac{1}{100}$ in the result in (i) above, determine the value of $\sqrt{23}$ correct to four places of decimals. (8 marks)

3. (a) Determine the values of a, b and c such that $3x^2 - 4x + 6 = a(x+b)^2 + c$. (7 marks)

(b) The sag l metres in a cable stretched between two supports, distance x meters apart, is given by $l \quad \frac{12}{x} + x$. Determine the distance between the supports when the sag is 20 metres. (5 marks)

(c) Use the method of elimination to determine the currents I_1 and I_2 flowing in the d.c. network of figure 1. (8 marks)

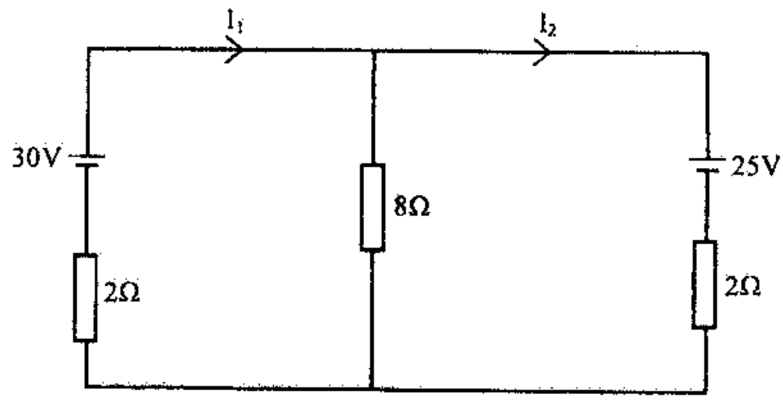

Fig.1

4. (a) Prove the identities:

(i) $\tan A + \cot A = 2 \operatorname{cosec} 2A$;

(ii) $\dfrac{Cos\, B + Cos\, C}{Sin\, B - Sin\, C} = Cot\left(\dfrac{B-C}{2}\right)$.

(6 marks)

(b) (i) Express $\sin 3\theta$ in terms of powers of $\sin \theta$.

(ii) By setting $\theta = \frac{\pi}{3}$ in the result in (i), determine the value of $\sin\left(\frac{\pi}{3}\right)$ without using a calculator, giving the answer in surd form.

(8 marks)

(c) Solve the equation $\cos\theta + \cos 3\theta + \cos 5\theta = 0$ for values of θ from 0° to 180° inclusive. (6 marks)

5. (a) Given that y = sin x, find $\dfrac{\delta y}{\delta x}$ from first principles. (5 marks)

(b) Find the first derivative of the function $f(x) = \sin\left(\dfrac{x-2}{x+3}\right)$. (6 marks)

(c) Locate the stationary points of the function $f(x) = x^3 + 3x^2 - 9x + 1$, and determine their nature. (9 marks)

6. (a) Evaluate the integrals:

(i) $\int_0^{\pi/2} (3\sin 2x + 2\cos x - 3)\delta x$;

(ii) $\int_0^1 \left(\dfrac{1}{\sqrt{x}} + x^2 - \dfrac{1}{x^{\frac{-3}{2}}}\right)\delta x$.

(7 marks)

(b) Sketch the region bounded by the curve $y = x^2$ and the straight line $y = 2 - x$, and find the area of the region. (7 marks)

(c) Use implicit differentiation to find the equation of the tangent to the curve:

$x^2 + y^2 + 3xy - 2x - 3y + 6$ at the point (1,1). (6 marks)

7. (a) Find the angle between the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{B} = -4\underset{\sim}{i} + 7\underset{\sim}{j} - 8\underset{\sim}{k}$. (7 marks)

(b) Determine the value of the scalar a, given that the three vectors, $\underset{\sim}{A} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + a\underset{\sim}{k}$ $\underset{\sim}{B} = 4\underset{\sim}{i} + 5\underset{\sim}{j} - 6\underset{\sim}{k}$ and $\underset{\sim}{C} = -2\underset{\sim}{i} + \underset{\sim}{j} + 8\underset{\sim}{k}$ are coplanar. (7 marks)

(c) Given the scalar field $\phi(x,y,z) = x^2y + zx + 3y^2$, determine at point (1, 2, 1) the:

(i) Grad ϕ;

(ii) directional derivative of ϕ in the direction of the vector $\underset{\sim}{A} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$.

(6 marks)

8. (a) Given $u(x,y) = x^2 - y^2 + 3x$, show that $\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta\ ^2}\quad 0$. (4 marks)

(b) Locate the stationary points of the function $z = x^2 + 3xy + y^2 - 2x - 3y + 10$, and determine their nature. (10 marks)

(c) Express $\sinh^{-1} x$ in logarithmic form, and hence determine the value of $\sinh^{-1} 2$, correct to three significant figures. (6 marks)