1521/203
1601/203
1602/203
MATHEMATICS II
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator.

This paper consists of ***EIGHT*** *questions.*

Answer any ***FIVE*** *questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Solve the equation $3^{2x+3} - 7(3^{x+1}) + 2 = 0$. (8 marks)

(b) Use the binomial theorem to expand $(2 + 3x)^{\frac{1}{2}}$ as far as the term in x^3, and state the range of values of x for which the expansion is valid. (4 marks)

(c) (i) Determine the first three terms in the binomial expansion of $(1 + 8x)^{\frac{1}{3}}$.

(ii) By putting $x = \frac{1}{27}$ in the result in (i), determine the value of $\sqrt[3]{35}$, correct to four decimal places. (8 marks)

2. (a) Given the matrices $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, determine $(AB)^{-1}$.

(10 marks)

(b) Three currents I_1, I_2 and I_3 in an electric circuit satisfy the equations:

$$I_1 - 2I_2 + I_3 = -6$$
$$-I_1 + 2I_2 + I_3 = 4$$
$$2I_1 - I_2 + I_3 = -2$$

Use Cramer's rule to determine the values of the currents. (10 marks)

3. (a) Given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{7}{25}$, and that A and B are acute, determine the values of:

(i) $\sin(A+B)$;
(ii) $\cos(A-B)$. (6 marks)

(b) Prove the identities:

(i) $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$;

(ii) $\dfrac{\cos B + \cos C}{\sin B - \sin C} = \cot\left(\dfrac{B-C}{2}\right)$. (7 marks)

(c) Solve the equation $2\cos 2\theta + 7\sin\theta = 5$, for values of θ between 0° and 180° inclusive. (7 marks)

4. (a) Find a unit vector that is perpendicular to the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{B} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 6\underset{\sim}{k}$. (7 marks)

(b) Given that the three vectors $\underset{\sim}{A} = \underset{\sim}{i} + \underset{\sim}{j} + a\underset{\sim}{k}$, $\underset{\sim}{B} = 7\underset{\sim}{i} + 2\underset{\sim}{j} + 0\underset{\sim}{k}$ and $\underset{\sim}{C} = 0\underset{\sim}{i} + \underset{\sim}{j} + 7\underset{\sim}{k}$ are coplanar, determine the value of a. (5 marks)

(c) Given the scalar function $\phi(x, y, z) = xy^3 + y^2 z$, determine, at the point (1, 1, -1):

(i) $\nabla\phi$

(ii) $\nabla \cdot \nabla\phi$

(iii) $\nabla \times \nabla\phi$. (8 marks)

5. (a) Given that the determinant:

$$\begin{vmatrix} -3 & 6 & -1 \\ -1 & 1-x & 0 \\ 3 & 0 & 1-x \end{vmatrix} = 0,$$ determine the possible values of x. (7 marks)

(b) Two resistors, when connected in parallel, gave a total resistance of 9.6 ohms. When connected in series, the total resistance was 40 ohms. If one of the resistances is R ohms,

(i) show that $R^2 - 40R + 384 = 0$;

(ii) determine the values of the resistors. (6 marks)

(c) Application of Kirchoff's laws to a resistive network yielded the simultaneous equations:

$$I_1 + I_2 - I_3 = 0$$
$$-I_1 + 2I_2 + I_3 = 9$$
$$2I_1 - I_2 + 3I_3 = 1$$

Use the substitution method to determine the values of the currents. (7 marks)

6. (a) Find $\frac{dy}{dx}$ from first principles, given that $y = \frac{1}{x+4}$. (5 marks)

(b) Use implicit differentiation to determine the equation of the:

(i) tangent;

(ii) normal

to the curve $9x + x^3y^2 - 2xy^2 + 3y = 6$, at the point (1, -1). (10 marks)

(c) The power developed in a resistor of resistance R ohms in series with an emf source of V volts with an internal resistance r ohms is given by $P = \frac{V^2 R}{(R+r)^2}$.

Show that the power developed is a maximum when R = r. (5 marks)

7. (a) Given $u = \frac{x+y}{x-y}$, show that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$. (9 marks)

(b) Locate the stationary points of the function $z = x^2 + 3y^2 - 4xy + 6x - 2y$ and determine their nature. (8 marks)

(c) Evaluate the integral $\int_0^{\frac{\pi}{2}} (3\sin 2x + \cos x - 2)\, dx$. (3 marks)

8. (a) Given the trigonometric identities:

(i) $\cos 2\theta = 1 - 2\sin^2\theta$;

(ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$;

(iii) $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$, use Osborne's rule to derive the corresponding hyperbolic identities. (3 marks)

(b) (i) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$;

(ii) Hence solve the equation $\cos 3\theta = \cos\theta$ for values of θ from 0° to 180° inclusive. (11 marks)

(c) Solve the equation $3\cosh\theta + 7\sinh\theta = 3$. (6 marks)

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